

Baryon-Meson Loop Effects on the Spectrum of Non Strange Baryons

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Corrections to the masses of baryons from baryon-meson loops can induce splittings between baryons which are comparable to those arising from the residual interactions between the quarks. These corrections are calculated using a pair-creation model to give the momentum-dependent vertices, and a model which includes configuration mixing to describe the wave functions of the baryons. A large set of baryon-meson intermediate states are employed, with all allowed $SU(3)_f$ combinations, and excitations of the intermediate baryon states up to and including the second band of negative-parity excited states. Roughly half of the splitting between the nucleon and Delta ground states arises from loop effects. The effects of such loops on the spectrum of negative-parity excited states are examined, and the resulting splittings are sensitive to configuration mixing caused by the residual interactions between the quarks. With reduced-strength one-gluon-exchange interactions between the quarks fit to the Delta-nucleon splitting, a comparison is made between model masses and the bare masses required to fit the masses of the states extracted from data analyses. This shows that it is necessary to also adjust the string tension or the quark mass to fit the splitting between the average bare masses of the ground states and negative-parity excited states, and that spin-orbit effects are likely to be important.

I. INTRODUCTION

In QCD there are $qqq(q\bar{q})$ configurations possible in baryons, and these must have an effect on the constituent quark model, similar to the effect of unquenching lattice QCD calculations. These effects can be modeled by allowing baryons to include baryon-meson (B^*M) intermediate states, which lead to baryon self energies and mixings of baryons of the same quantum numbers. A calculation of these effects requires a model of baryon-baryon-meson (BB^*M) vertices and their momentum dependence. It is also necessary to have a model of the spectrum and structure of baryon states, including states not seen in analyses of experimental data, in order to provide wave functions for calculating the vertices, and to know the thresholds associated with intermediate states containing missing baryons.

Baryon self energies due to B^*M intermediate states and B^*M decay widths can be found from the real and imaginary parts of loop diagrams. The size of such self energies can be expected to be comparable to baryon widths. For this reason, they cannot be ignored when comparing the predictions of any quark model with the results of analyses of experiments. Since the mass splittings between states which result from differences in self energies are likely similar to those that arise from the residual interactions between the quarks (defined to be interactions which are present after taking into account confinement), a self-consistent calculation of the spectrum needs to adjust the residual interactions, and so the wave functions of the states used to calculate the BB^*M vertices, to account for these additional splittings. In time-ordered perturbation theory (TOPT), the contribution to the self-energy of a baryon B with bare energy E from the baryon-meson loop illustrated in Figure 1 is

$$\Sigma_{B^*M}^B(E) = \int dk \frac{\mathcal{M}_{BB^*M}^\dagger(k) \mathcal{M}_{BB^*M}(k)}{E - \sqrt{m_{B^*}^2 + k^2} - \sqrt{m_M^2 + k^2}}, \quad (1)$$

where the calculation is carried out in the center-of-momentum frame of the initial baryon. Note that the intermediate baryon and meson are assigned their physical masses m_{B^*} and m_M . This ensures [1] that the poles due to decay thresholds are in their correct positions [2].

The strong decay matrix element $\mathcal{M}_{BB^*M}(k)$ depends on the loop momentum and the spin, flavor and spatial structure of the hadrons involved, and plays the role of a form factor. When the effects of confinement are included in the spatial structure of the hadrons, the factor $[\mathcal{M}_{BB^*M}(k)]^2$ has the effect of suppressing high-momentum contributions to the loop and rendering it finite. Care has to be taken to evaluate the principal part of the loop integral when it crosses a pole, present if the bare energy E is sufficiently large to allow the decay $B \rightarrow B^*M$ to proceed. The imaginary part of the loop integral is given by the residue of this pole, and is related to the partial width for this

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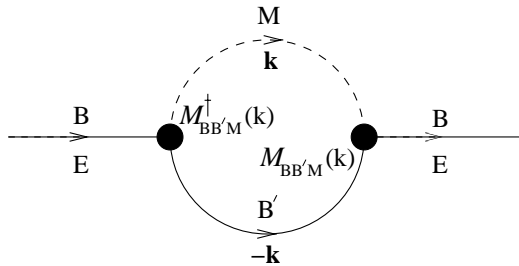


FIG. 1: Baryon-meson $B'M$ loop contributing to the energy-dependent self-energy $\Sigma_B(E)$ of a baryon B with bare mass E , evaluated in the rest frame of the initial baryon. The intermediate baryon and meson are assigned physical masses m'_B and m_M .

decay. A model of these strong-decay vertices can, therefore, be fit to the physical decay rates, and can then be used to predict the variation of the strong decay matrix element $\mathcal{M}_{BB'M}(k)$ away from the physical decay point.

The self energy of a baryon B is then evaluated by adding the contributions from all possible intermediate loops

$$\Sigma_B(E) = \sum_{B'M} \int d\mathbf{k} \frac{\mathcal{M}_{BB'M}^\dagger(k) \mathcal{M}_{BB'M}(k)}{E - \sqrt{m_{B'}^2 + k^2} - \sqrt{m_M^2 + k^2}} \quad (2)$$

It is crucial that this sum is over a set of intermediate states which is large enough that the differences in the self energies (the self energies themselves are not observables) do not change appreciably with the inclusion of additional states. This is a non-trivial requirement, especially when the intermediate state includes excited baryons, since the baryon spectrum includes a large number of excited states close in energy to the ground states. Not only does the sum have to include (ground state) baryons and mesons of different flavors and total quark spins, it also in principal should include spatial excitations of both the baryons and mesons. For larger hadron J values, more than one relative angular momentum of the intermediate hadrons is possible. The resulting complexity has often led in previous calculations to premature truncation of the sum in Eq. (2).

Ignoring, for the moment, spatial excitations of the intermediate hadrons, it is possible to define a complete set of spin-flavor symmetry-related intermediate states. Consider the effects of baryon-meson intermediate states on the Δ - N mass splitting, traditionally used to set the strength of the spin-dependent contact interactions between the quarks. If these spin-dependent interactions are turned off, it is still possible that these states have self energies from $B'M$ loops which are different, and so cause a splitting between the states. Assume for now that there are only ground-state baryons and mesons, made up of u , d , and s quarks with the same mass, and that there are no interactions between the quarks other than confinement. In this $SU(6)$ -symmetry limit all (ground state) baryons will have the same masses and wave functions, and the same is true of mesons. All of the intermediate states $B'M$ used to calculate baryon self energies will have the same mass $m_{B'} + m_M$, and the energy-dependent self energies in Eq. (2) will differ only because of the flavor and spin structure of the strong decay matrix elements $\mathcal{M}_{BB'M}(k)$.

It should be true that in this limit all ground state baryon self energies are the same, and this has been demonstrated to be true in Ref. [3], *if* the set of intermediate states includes a complete set of spin-flavor [$SU(6)$] symmetry related allowed combinations of the ground state octet and decuplet baryons, and ground state pseudoscalar and vector mesons. In particular, the ground-state Δ and nucleon states will be degenerate in this limit only if non strange and strange pseudoscalar and vector mesons are included in the intermediate states. This implies that calculations of self energies which do not include vector mesons, for example, do not start at this limit and so cannot be expected to produce physically meaningful results away from it.

With the exception of the work of Ref. [4], previous calculations of the self energies of ground state and negative-parity excited baryons use baryon-meson intermediate states including only baryon (spatial) ground states [1, 3, 5, 6]. However, the importance of including spatially-excited baryon states has been established in a calculation [4] of the Δ - N splitting. Here the intermediate states are restricted to the set of states $B'\pi$, with baryon states B' chosen from a set of ground and excited N and Δ states. The sum over intermediate states is shown to converge to a stable result for the Δ - N splitting only when excited states from the $N = 0$ (ground), $N = 1$ (lowest-lying negative-parity excited), $N = 2$ (positive-parity excited), and $N = 3$ (highly-excited negative-parity) bands of states are included. Similar results for the convergence properties of the ρ and ω meson self energies are found in a calculation of the effects of meson-meson intermediate states on meson masses [7]. This suggests that calculations of baryon self energies which restrict the intermediate baryons to (spatial) ground states cannot be expected to have converged.

In principle it may also be necessary to check for convergence of a sum over spatially-excited intermediate meson states. However, since (i) orbital and radial excitations of mesons tend to be significantly more massive than their

corresponding ground states, and (ii) introducing spatial excitation into the meson wave function generally reduces the size of the $BB'M$ vertices, and (iii) the multiplicity of spatially-excited meson states is much lower than that of spatially-excited baryon states, it may be possible to ignore spatially-excited mesons in the set of intermediate states. This is the approach adopted here.

Interestingly, in the model of Ref. [4], the difference in the pionic self energies of the odd-parity excited states and the ground state converges too slowly to make definite conclusions. This may be due to the choice of $BB'M$ amplitudes, where pions are emitted directly from the quarks with a (non relativistic) pseudoscalar coupling, and an additional (somewhat hard) axial form factor

$$F_\pi(\mathbf{k}^2) = 1/(1 + \mathbf{k}^2/\Lambda_\pi^2), \quad (3)$$

with $\Lambda_\pi = 1275$ MeV corresponding to the mass of the a_1 meson. Since the loop integrals involve elementary intermediate pions, a factor of $1/\omega_k$ is included, where $\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}$ is the pion energy, from the normalization of the wave function of the intermediate pion. Note that this factor is not present in the pion center of mass wave function in non relativistic models which treat it as a composite particle. Although the presence of this factor has the effect of further suppression of high-momentum contributions to the integral over the loop momentum [1], the net result is an effective pion-nucleon vertex which is probably too hard. The same is likely to be true of the work of Ref. [5], which also uses an elementary-meson emission strong-decay model for the $BB'M$ vertices. In subsequent models and the present work a more rapid decrease of the vertex amplitudes with k^2 is shown to produce better results for the mass shifts, and this can be attributed to an effective size for the operator which creates a constituent quark-antiquark pair [1, 7]. The issue of the poor convergence of the sum over intermediate excited baryons found in Ref. [4] of the self energies of the negative-parity excited states will be resolved in the present work.

This illustrates the importance of the choice of model to describe the strong vertex amplitudes $\mathcal{M}_{BB'M}(k)$. In particular, it is necessary to take into account the spatial structure of the emitted mesons to avoid vertex amplitudes which do not sufficiently suppress the contributions of the $B'M$ loops at large relative momentum between the two hadrons. At the same time, the vertex amplitudes should contain information about the spatial structure of the initial and intermediate hadron states. A popular phenomenological strong-decay model based on the creation of a $q\bar{q}$ pair with vacuum (3P_0) quantum numbers and applied to a baryon and meson strong decays [8] has been adopted in previous calculations of loop effects [1, 3]. In the calculations of Refs. [1, 3], a single baryon radius and meson radius and unmixed harmonic-oscillator wave functions were used to describe the baryon states, which is equivalent to assuming SU(6) symmetry in the wave functions. These approximations are not made in the present work. A more sophisticated decay model, similar to that developed for mesons in Ref. [9], was adopted in the work of Ref. [6]. This work also uses antisymmetrized $(3q)(q\bar{q})$ cluster-model wave functions composed of simple harmonic oscillator wave functions and plane-wave relative motion to describe the baryon-meson intermediate states.

An important goal of prior calculations of the mass splittings of negative-parity excited baryon arising from self energies is a possible resolution of the spin-orbit problem in baryons. In general, spin-orbit effects are too large when calculated with one-gluon exchange residual interactions with the strength required to cause all of the contact splittings between ground states and negative-parity excited states evident in the spectrum. The situation is different when the residual interactions between the quarks are assumed to arise from one-boson exchange, as there are no corresponding spin-orbit interactions, but those arising from Thomas precession in the confining potential should still be present and will not be negligibly small [10, 11]. With the introduction of mass splittings due to loop effects it is possible [3, 4] to use a reduced-strength residual interaction (in one-gluon exchange models this means a smaller value of α_s in the limit of low Q^2), which will naturally reduce the size of the resulting spin-orbit effects [3].

A second possibility is that, in addition to reducing the strength of the residual interactions required to fit the observed splittings, it may happen that the self energies induce splittings between the bare masses which resemble spin-orbit effects. Ref. [1] explores the possibility that any spin-orbit splittings in the spectrum of low-lying negative-parity excited N , Δ , Λ and Σ states arises from the effects of differences in the self energies. It may also be possible that the spin-orbit splittings due to differences in the self energies are opposite in sign to those expected from one-gluon-exchange [5]. This intriguing possibility has been explored in Ref. [6]. The results show that it may be possible to arrange a cancellation between spin-orbit splittings arising from the interactions between the quarks and from loop effects, and to describe the mixings and decay widths of these states in the same model. Notable exceptions are the flavor singlet (lowest lying) negative-parity Λ states $\Lambda(1405)$ and $\Lambda(1520)$ which are about 100 MeV too heavy, as in simple three-quark models.

Conclusions made in the models described above about spin-orbit forces in negative-parity excited baryons are likely to be premature, given the information provided about convergence in Ref. [4]. It is shown in the present work that the inclusion of negative-parity excited baryons as intermediate states is crucial to the accurate calculation of the mass shifts of these states.

It is, therefore, clear from considering prior work that a self-consistent model of baryon self energies must employ a full set of spin-flavor [SU(6)] symmetry related $B'M$ intermediate states, and at the same time must include excited

baryon states up to at least the $N = 3$ band in order for the sum over intermediate state baryons to have converged. This requires a detailed and universal decay model, such as the 3P_0 pair-creation model, which is capable of relating the baryon spectrum and the amplitudes for decay of a wide variety of baryon initial states to a wide variety of baryon-meson final states in an efficient way. The decay model needs to take into account the spatial structure of the intermediate meson. It is also clear that it will be necessary to modify the usual momentum dependence of the decay amplitudes calculated in this model to take into account the size of the constituent quark-pair creation vertex.

In addition, the size of these loop effects requires that the interactions between the quarks required to fit the observed spectrum be changed by their presence. To be consistent the wave functions used to calculate the vertex amplitudes should then also be changed, and the effect of these changes on the self-energies examined. The work of Ref. [4] has showed that the Δ -nucleon splitting may not be sensitive to such details, but from the sensitivity to the details of the inter-quark Hamiltonian used to describe the hadron states observed in many of these calculations, it can be expected that this will be an important effect in the calculation of the self energies of negative-parity excited baryons.

II. BARYON SELF ENERGIES AND BARE ENERGIES

In the present work, a calculation of the self energies of ground and negative-parity excited N and Δ states is carried out using a 3P_0 pair-creation model to calculate the momentum-dependent vertices $\mathcal{M}_{BB'M}(k)$, with wave functions calculated using a relativized model [12] with a variable-strength spin-dependent (one-gluon exchange) contact interaction between the quarks. This calculation takes into account a full set of spin-flavor symmetry related intermediate states $B'M$ with

$$\begin{aligned} M &\in \{\pi, K, \eta, \eta', \rho, \omega, K^*\} \\ B' &\in \{N, \Delta, N^*, \Delta^*, \Lambda, \Sigma, \Lambda^*, \Sigma^*\}, \end{aligned} \quad (4)$$

including all excitations of all of the intermediate baryon states up to and including $N = 3$ band states. Note that ϕ mesons couple weakly to non-strange baryon states since such decays are OZI suppressed, and so they are ignored.

The usual version of the 3P_0 model gives vertices that are too hard, and the loop integrals required to evaluate the self energies get large contributions from high momenta. Here these vertices are modified by adopting a pair-creation operator used in previous calculations of loop effects in mesons [7] and baryons [1]. This operator includes a form factor $\exp(-f^2[\mathbf{p}_q - \mathbf{p}_{\bar{q}}]^2)$ with $f^2 = 2.8 \text{ GeV}^{-2}$, which gives the quark-pair-creation vertex a size of around 0.33 fm. As the self energies due to a given intermediate state depend crucially on the masses adopted for the intermediate hadrons, these are taken to be the physical masses, where known, and model masses [12] otherwise.

Since the self energies $\Sigma_B(E)$ calculated using Eq. (2) are energy-dependent, it is necessary to solve for the ‘bare’ mass E_B^0 required to reproduce the known physical masses m_B of a given baryon B by solving self-consistent equations

$$E_B + \Sigma_B(E_B) = m_B. \quad (5)$$

This requires knowledge of the self energies at a range of bare energies. For example, to examine the $\Delta - N$ splitting, the ‘bare’ masses required to reproduce the known physical masses of the N and Δ are found by solving a pair of (uncoupled) self-consistent equations

$$E_N + \Sigma_N(E_N) = m_N, \quad E_\Delta + \Sigma_\Delta(E_\Delta) = m_\Delta \quad (6)$$

for the ‘bare’ masses E_N^0 and E_Δ^0 . Note that the self energies tend to be large and negative, but only differences in the self energies are observable.

Details of how the calculation is made are given in Sec. III. Results for the self energies and resulting bare energies of ground state and negative-parity excited state non-strange baryons are given in Secs. IV, V and VI. The conclusions of this study are given in Sec. VII.

III. BARYON-MESON INTERMEDIATE STATE CONTRIBUTIONS

In this section some of the formalism necessary to describe the effects of baryon-meson loops on baryon masses is presented.

A. Strong decay vertices

A key ingredient of this calculation is the form of the momentum dependence of the baryon-baryon-meson vertices. Here the 3P_0 pair-creation strong decay model is used to obtain the structure of each vertex and hence its momentum dependence. The modified pair-creation operator has the form

$$\begin{aligned}
 T = & -3\gamma \sum_{i,j} \int d\mathbf{p}_i d\mathbf{p}_j \delta(\mathbf{p}_i + \mathbf{p}_j) C_{ij} F_{ij} e^{-f^2(\mathbf{p}_i - \mathbf{p}_j)^2} \\
 & \times \sum_m \langle 1, m; 1, -m | 0, 0 \rangle \\
 & \times \chi_{ij}^m \mathcal{Y}_1^{-m}(\mathbf{p}_i - \mathbf{p}_j) b_i^\dagger(\mathbf{p}_i) d_j^\dagger(\mathbf{p}_j),
 \end{aligned} \tag{7}$$

where C_{ij} and F_{ij} are the color and flavor wave functions of the created pair, both assumed to be singlet, χ_{ij} is the spin triplet wave function of the pair, and $\mathcal{Y}_1(\mathbf{p}_i - \mathbf{p}_j)$ is the solid harmonic indicating that the pair is in a relative P -wave ($l = 1$). Note that the threshold behavior resulting from the $|\mathbf{p}_i - \mathbf{p}_j|$ factor in the solid harmonic is as seen experimentally. Here $b_i^\dagger(\mathbf{p}_i)$ and $d_j^\dagger(\mathbf{p}_j)$ are the creation operators for a quark and an antiquark with momenta \mathbf{p}_i and \mathbf{p}_j , respectively. As mentioned above, the additional exponential has been introduced to give the vertex a spatial extent by creating the quark-antiquark pair over a smeared region, instead of at a point as is the case in the usual version of the model.

Baryon wave functions which result from diagonalizing the model qqq Hamiltonian described in Sec. III C in a large harmonic oscillator basis (up to and including the $N = 7$ oscillator band) are used along with single-oscillator meson wave functions to evaluate the transition matrix elements of the pair-creation operator in Eq. (7). There are only two phenomenological parameters in this model. These are γ , the 3P_0 coupling strength, which is fitted to the experimentally well known $\Delta \rightarrow N\pi$ decay, and f , which is set to give a reasonable quark-pair-creation vertex size. A similar model using the same wave functions but [13] with $f = 0$ has been tested against a large number of measured baryon decays [14].

For the transition $B \rightarrow B'M$, we are interested in evaluating the transition amplitude

$$A_{B \rightarrow B'M} = \langle B'M | T | B \rangle, \tag{8}$$

which is given in Appendix A. The notation illustrated in Fig. 2 was used to arrive at this form. Note that the decaying baryon is assumed to be at rest and that the relative momentum of the final baryon and meson is \mathbf{k}_0 .

Given the very large number of loops which can contribute to the self-energy of a given baryon [15], and the requirement of a calculation of the momentum dependent vertex $\mathcal{M}_{BB'M}(k)$ for each of them, this is a necessarily computationally intensive calculation. Code has been written in *Maple*, and executed on a computer cluster, which analytically calculates the matrix elements of the operator in Eq. (7) for each pair of oscillator substates involved in the external and intermediate baryon wave functions. This has the advantage that the momentum-dependent vertices can then be repeatedly projected out of these stored matrices with the external and intermediate baryon wave functions, which change as the model qqq Hamiltonian is adjusted. This process is described in what follows.

B. Self-consistent baryon mass calculation

Given the expected size of splittings arising from self energies, it will be necessary to adjust the qqq Hamiltonian to fit the spectrum of bare energies which result from fitting to baryon masses extracted from analyses of scattering data. Each variation of the Hamiltonian will produce a new set of baryon wave functions, which can in turn be used to calculate the momentum dependence of the $BB'M$ vertices. These will yield new self energies, from which a new spectrum of bare masses required to fit the baryon masses from analyses of data can be obtained [*i.e.* solutions of Eq. (5)]. Obviously an iterative procedure will be required to find the form of the qqq Hamiltonian which best fits the physical masses.

Given the complexity of the self-energy calculations, full implementation of this iterative procedure is postponed until the set of external states is expanded to include strange baryons. Nevertheless, important conclusions about the strength of the contact part of the spin-spin interaction will be made below by comparison of the spectrum of a model qqq Hamiltonian and the consistently calculated bare energies. In addition, the effects of configuration mixing due to a tensor interaction are explored in Section V. In what follows, the form of the model qqq Hamiltonian used in the present calculations is described.

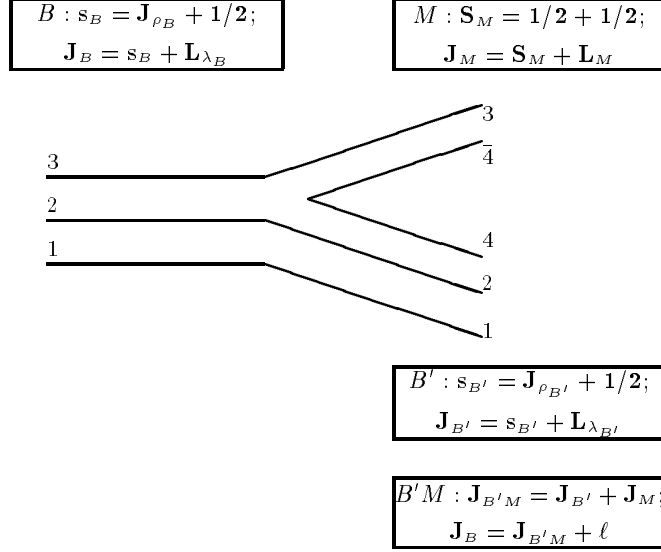


FIG. 2: Schematic diagram of the decay $B \rightarrow B' M$ in the 3P_0 model. The angular momentum notation is shown. The decay proceeds through $B(123) \rightarrow 12(44)3 \rightarrow B'(124)M(43)$.

C. Model qqq Hamiltonian

In order to calculate the strength of the baryon-baryon-meson (BB'M) vertices used here we require a model of the spectrum and wave functions of the external (B) and intermediate-state (B') baryons. While it may be possible to use physical masses for those states present in analyses of scattering data, a model is required to estimate the masses of states missing in these analyses but present when baryons are composed of three quarks treated symmetrically.

It is also inconsistent to use physical masses for the intermediate states along with harmonic-oscillator wave functions. The spectrum is closer to that of a linear potential, and the wave functions will include significant mixing due to this if expressed in a harmonic-oscillator basis. After taking confinement into account by means of a linear potential, most models have some sort of short-range residual interaction between the quarks. It is interesting to explore whether the self-energies of baryons depend on the presence of such residual interactions.

In order to provide explicit wave functions roughly consistent with the spectrum of the intermediate states, a model qqq Hamiltonian with the relativistic form of the kinetic energy and a linear confinement potential

$$H_0 = \sum_i \sqrt{p_i^2 + m_i^2} + F \sum_{i < j} b r_{ij} \quad (9)$$

is used here, where $b = 0.15 \text{ GeV}^2$ is the baryon string tension and $F = 0.55$ is chosen to minimize the difference between this and the sum of the lengths of a Y-shaped string [16] in a spherical ground state (for details see Ref. [12]). Note $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between quarks i and j .

In order to explore the sensitivity of the self energies (or the bare energies required to fit the physical masses extracted from analyses of scattering data) to the presence of residual interactions, H_0 is supplemented by pairwise Coulomb and contact interactions of a form motivated by one-gluon exchange

$$H_{\text{Coulomb}} + H_{\text{contact}} = \sum_{i < j} H_{ij}, \quad (10)$$

where

$$H_{ij} = (\beta_{ij})^{1/2 + \epsilon_{\text{Coul}}} \tilde{G}(r_{ij}) (\beta_{ij})^{1/2 + \epsilon_{\text{Coul}}} + (\delta_{ij})^{1/2 + \epsilon_{\text{cont}}} \left[\frac{2\mathbf{S}_i \cdot \mathbf{S}_j}{3m_i m_j} \nabla^2 \tilde{G}(r_{ij}) \right] (\delta_{ij})^{1/2 + \epsilon_{\text{cont}}}. \quad (11)$$

The outer factors of powers of

$$\delta_{ij} = m_i m_j / \left(\sqrt{p_{ij}^2 + m_i^2} \sqrt{p_{ij}^2 + m_j^2} \right) \quad (12)$$

and

$$\beta_{ij} = 1 + p_{ij}^2 / \left(\sqrt{p_{ij}^2 + m_i^2} \sqrt{p_{ij}^2 + m_j^2} \right), \quad (13)$$

where p_{ij} is the magnitude of the momentum of the interacting quarks in their center-of-momentum frame, are momentum-dependent relativistic factors designed to parameterize the dependence of these potentials away from the non relativistic limit, and are based on the momentum dependence of the one-gluon exchange T -matrix element between free quarks, where $\epsilon_{\text{cont}} = \epsilon_{\text{Coul}} = 0$. For bound quarks this momentum dependence will in general be modified. In the following, $\epsilon_{\text{cont}} = -0.168$ as in Ref. [12], and ϵ_{Coul} is given the same value for simplicity.

In Eq. (11)

$$\tilde{G}(r_{ij}) = - \sum_k \frac{2\alpha_k}{3r_{ij}} \text{erf}(\sigma_{kij} r_{ij}) \quad (14)$$

is a Coulomb potential smeared over a Gaussian distribution

$$\rho_{ij}(\mathbf{r}_{ij}) = \frac{\sigma_{ij}^2}{\pi^{3/2}} e^{-\sigma_{ij}^2 r_{ij}^2} \quad (15)$$

of the inter-quark coordinate, with smearing parameters σ_{ij} which depend on the masses of the interacting quarks via (for details see Ref. [12])

$$\sigma_{ij}^2 = \frac{\sigma_0^2}{2} \left\{ 1 + \left[\frac{4m_i m_j}{(m_i + m_j)^2} \right]^4 \right\} + s^2 \left[\frac{2m_i m_j}{m_i + m_j} \right]^2, \quad (16)$$

where σ_0 and s are universal parameters. Note also that the strong coupling runs according to the usual perturbative formula at large Q^2 , and saturates to a value $\alpha_s^{\text{critical}}$ at low Q^2 . This behavior is fit to a functional form

$$\alpha_s(Q^2) = \sum_{k=1}^3 \alpha_k e^{-Q^2/4\gamma_k^2} \quad (17)$$

and the parameters σ_{kij} in Eq. (14) have values given by

$$1/\sigma_{kij}^2 = 1/\gamma_k^2 + 1/\sigma_{ij}^2. \quad (18)$$

In Ref. [12] the smearing parameters relevant to this work were $\sigma_{uu} = \sigma_{ud} = \sigma_{dd} = 1.832$ GeV, and $\sigma_{us} = \sigma_{ds} = 1.702$ GeV, resulting from $\sigma_0 = 1.8$ GeV and $s = 1.55$. This results in an effective ‘size’ of a constituent quark of roughly 0.08 fm. In related work it has been shown that, in order to fit the nucleon electromagnetic form factors in a light-cone based model using the wave functions which result from the Hamiltonian used in Ref. [12], form factors for the constituent quarks are required which give them an electromagnetic size much larger than this [17]. This is because the contact interaction of Eq. (11), which is a smeared Dirac δ function, has most of its strength at short range, and so builds substantial high-momentum components into the wave functions of states like the nucleon with net attractive contact interactions. A constituent quark form factor which falls off rapidly with momentum transfer is required to compensate. A simple solution to this apparent mismatch of the strong and electromagnetic sizes of the constituent quark, which is adopted here, is to use a parameter $\sigma_0 = 0.83$ GeV, which gives the up and down quark interactions a smearing $\sigma = 0.9$ GeV, or a constituent quark size of roughly 0.15 fm.

Accompanying this increase in the effective range of the contact interaction is a reduction in its average strength, which can be quantified by the splitting caused by H_{contact} between the ground-state Δ and nucleon. As a consequence, if the value $\alpha_s^{\text{critical}} = 0.60$ from Ref. [12] is used with this longer-range contact interaction, the size of the $\Delta - N$ splitting due to H_{contact} is reduced by about a factor of two. In what follows a value $\alpha_s^{\text{critical}} = 0.55$ was used whenever $H_{\text{Coulomb}} + H_{\text{contact}}$ was included in the model qqq Hamiltonian.

Tensor interactions between the quarks are also considered, in order to examine the effects of configuration mixing in the wave functions of spin-partner states such as $N1/2^-(1535)$ and $N1/2^-(1650)$, which have quark spin 1/2 and 3/2 respectively when the interactions between quarks are overall spin scalars (such as H_{contact}). These can be consistently included by adding an interaction

TABLE I: Representative splittings in MeV of bare energies as a function of the maximum level N_{\max} of excitation of the intermediate baryons B' , calculated using $\alpha = 0.5$ GeV and with Coulomb and contact interactions only in H_{qq} .

	$N_{\max} = 0$	$N_{\max} = 1$	$N_{\max} = 2$	$N_{\max} = 3$
$E_{\Delta}^0 - E_N^0$	315	152	153	155
$E_{1650}^0 - E_{1535}^0$	33	363	-25	-25
$E_{1700}^0 - E_{1520}^0$	25	280	-17	-25
P -wave-grd. state	-119	-49	339	334

$$H_{\text{tensor}} = \sum_{i < j} (\delta_{ij})^{1/2 + \epsilon_{\text{tens}}} \frac{1}{3m_i m_j} \left[\frac{3\mathbf{S}_i \cdot \mathbf{r}_{ij} \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \left[\frac{1}{r_{ij}} \frac{d\tilde{G}(r_{ij})}{dr_{ij}} - \frac{d^2\tilde{G}(r_{ij})}{dr_{ij}^2} \right] (\delta_{ij})^{1/2 + \epsilon_{\text{tens}}}, \quad (19)$$

and in what follows we have chosen ϵ_{tens} to be the same as ϵ_{cont} and ϵ_{Coul} for simplicity.

IV. Δ -NUCLEON SPLITTING

The Δ -Nucleon splitting has been used by those constructing models of the baryon spectrum to determine the strength of the short-range interactions between the quarks. It is therefore of considerable interest to examine whether this splitting is modified by the self energies which result from the presence of $B'M$ intermediate states. Prior calculations show a substantial splitting between the bare energies required to fit the physical Δ and nucleon masses [3, 4, 5], but these calculations may not have converged due to the restriction of the intermediate states to ground state baryons [3, 5], or to non-strange baryons and pions [4]. Here this splitting is re-examined without these restrictions, using wave functions generated by the Hamiltonian H_0 of Eq. (9) without residual interactions between the quarks, and also those found using the reduced strength one-gluon exchange interaction (with and without consistent tensor interactions and the configuration mixing they cause) described in Sec. III C. It is of particular interest to see whether the sum over intermediate states in Eq. (2) has converged to a stable Δ -Nucleon bare mass splitting, and whether this splitting depends on the nature of these residual interactions, as seen in Ref. [4].

When baryon wave functions resulting from the qqq Hamiltonian H_0 (only confining interactions between the quarks) are used to describe the full set of $B'M$ intermediate states in Eq. (4), including excited baryon states up to the $N = 3$ band, this results in bare masses which satisfy $E_{\Delta}^0 - E_N^0 \simeq 150$ MeV. The addition of the reduced strength Coulomb and contact interactions described in Section III C changes this slightly to roughly 155 MeV. The results described below use the latter Hamiltonian except where noted. Figures 3 and 4 illustrate the energy dependence of the self energy of the nucleon and Δ ground states, by plotting $E + \Sigma(E)$ against E . Eqs. (6) are solved where the curves intersect the horizontal solid lines at $m_N = 938$ MeV and $m_{\Delta} = 1232$ MeV. The four curves show the effects of increasing the maximum level of excitation of the intermediate baryons from ground states ($N_{\max} = 0$), to $N_{\max} = 3$, with regions of rapid curvature corresponding to various decay thresholds.

The first row of Table I illustrates the dependence of the difference in the bare energies $E_{\Delta}^0 - E_N^0$ which solve Equations (6) on the maximum level of excitation of the intermediate baryon states. This and Figs. 3 and 4 show the importance of the inclusion of intermediate states involving “ $N = 1$ band” negative-parity excited baryons, i.e. those which have wave functions predominantly made up of $N = 1$ band oscillator substates. These results confirm those of Ref. [4], where it is shown, in a model with the intermediate states restricted to pions and ground and excited states of N and Δ , that the inclusion of excited baryon intermediate states substantially reduces the $\Delta - N$ splitting.

In the present work intermediate states involving $N = 2$ and $N = 3$ band excited baryon states are relatively unimportant, contributing less than 10 MeV to the splitting. These results clearly demonstrate that, by $N_{\max} = 3$, the difference of the self energies of the Δ and nucleon has converged, as seen in the quite different model of these effects in Ref. [4]. Note that this convergence occurs even with the large increase in the multiplicity of intermediate baryon states at higher N values.

Table II shows the difference of the self energies of the Δ and nucleon, $\Sigma_{\Delta}(E_{\Delta}^0) - \Sigma_N(E_N^0)$, broken up into contributions from intermediate states of different flavor and level of baryon excitation. It is important to note that all of these (energy-dependent) self-energy differences are evaluated at the bare energies which solve Eqs. (6) with the complete sum of SU(6)-related intermediate states including baryon excitations up to $N=3$. Their sum is therefore the difference of the physical $\Delta - N \simeq 295$ MeV splitting and the $N_{\max} = 3$ result from Table I. As these differences are strongly energy dependent and the bare energies depend on the maximum level of excitation of the

TABLE II: Contributions to the difference $\Sigma_{\Delta}(E_{\Delta}^0) - \Sigma_N(E_N^0)$ in MeV of the self energies of the Δ and nucleon, evaluated at the full bare energies E_{Δ}^0 and E_N^0 . Self energies are calculated using $\alpha = 0.5$ GeV and with Coulomb and contact interactions only in H_{qqq} . Columns correspond to the excitation level of the intermediate baryons, and the row labeled π includes contributions from $N^*\pi$ and $\Delta^*\pi$ intermediate states, that labeled K includes contributions from Λ^*K and Σ^*K intermediate states, etc.

	$N = 0$	$N = 1$	$N = 2$	$N = 3$	total
π	-24	55	7	113	151
K	-4	27	12	-25	10
η	9	-14	-4	-13	-22
η'	22	26	-3	-3	42
ρ	-17	333	-16	-76	224
ω	87	130	-10	-23	184
K^*	175	-569	-34	-24	-452
total	248	-12	-48	-51	137

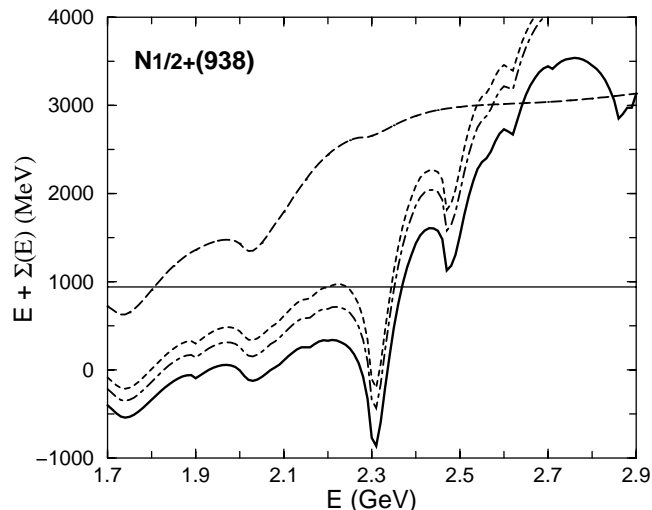


FIG. 3: Sum of bare and self energies for the ground-state nucleon, for $\alpha = 0.5$ GeV and with Coulomb and contact interactions only in H_{qqq} . The long-dashed curve is calculated with only ground state intermediate baryons B' , the short-dashed curve adds $N = 1$ band baryons, the short-dashed long-dashed curve adds $N = 2$ baryons, and the solid curve adds $N = 3$ intermediate baryons. The first of Eqs. (6) is solved where the curves intersect the horizontal solid line at $m_N = 938$ MeV.

intermediate-state baryons, the convergence of the $\Delta - N$ splitting is demonstrated by reading Table I from left to right, *not* Table II.

What is clear from Table II is that intermediate $B'\pi$ states (with B' taken from ground and excited states of the nucleon and Δ) will contribute a self-energy difference only a little larger than the full result. Intermediate states involving other pseudoscalar mesons add another 30 MeV, and the additional terms due to intermediate states involving all of the vector mesons ρ , ω and K^* reduce the sum by 50 MeV. Intermediate states involving ground and excited state baryons and vector mesons are clearly important. Interestingly, although the self-energy difference due to sets of intermediate states involving ρ , ω and K^* mesons are individually large, especially when accompanied by $N = 1$ band baryons, their sum is not. This is reminiscent of results for $\rho - \omega$ splitting in the very similar model of Ref. [7], where it was shown that there are meson-meson intermediate states which give large contributions to the splitting, but which largely cancel when considered in certain groups.

Similar conclusions resulted from a prior calculation which did not use a full set of SU(6)-related $B'M$ intermediate states, where substantial $E_{\Delta}^0 - E_N^0$ splittings were found using wave functions including reduced-strength one-gluon exchange interactions [4]. In addition, this agrees reasonably well with the expectation from a model with both one-pion exchange and one-gluon exchange residual interactions between the quarks [18], that about two thirds of the Δ - N splitting comes from one-gluon exchange interactions. However, in the present picture the rest of the splitting arises from a source very different from one-boson-exchange (OBE) or similar mechanisms between the quarks. Note that the self-energy diagrams evaluated in this work include meson-exchange interactions between the quarks, and also deal consistently with quark self energies from meson loops and the threshold effects of a large number of $B'M$ intermediate states.

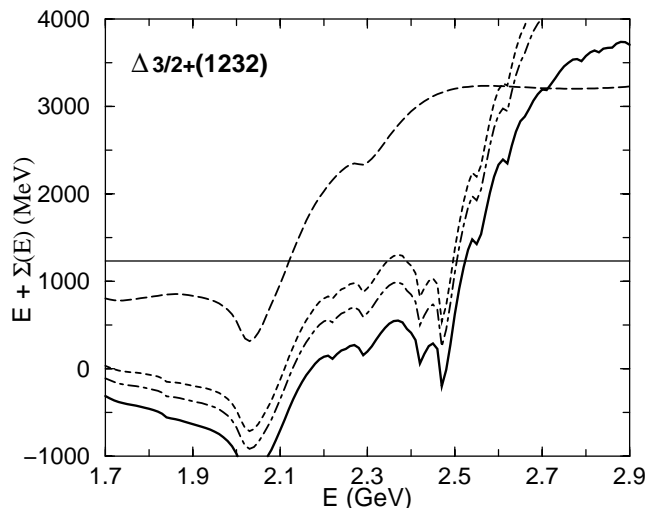


FIG. 4: Sum of bare and self energies for the ground-state Δ , for $\alpha = 0.5$ GeV and with Coulomb and contact interactions only in H_{qqq} . Curves are labeled as in Fig. 3. The second of Eqs. (6) is solved where the curves intersect the horizontal solid line at $m_\Delta = 1232$ MeV.

TABLE III: Splitting $E_\Delta^0 - E_N^0$ in MeV of the bare Δ and nucleon energies calculated using wave functions with H_{qqq} equal to H_0 (with $\alpha = 0.4$ GeV), $H_0 + H_{\text{Coulomb}} + H_{\text{contact}}$, and $H_0 + H_{\text{Coulomb}} + H_{\text{contact}} + H_{\text{tensor}}$ (with $\alpha = 0.5$ GeV), and a full set of intermediate states with excited baryons up to $N_{\text{max}} = 3$.

H_{qqq}	H_0	$H_0 + H_{\text{Coulomb}} + H_{\text{contact}}$	$H_0 + H_{\text{Coulomb}} + H_{\text{contact}} + H_{\text{tensor}}$
$E_\Delta^0 - E_N^0$	151	155	145

Table III illustrates that the introduction of residual interactions which can accommodate the rest of the observed Δ - N splitting affects the wave functions and so the vertices, but does not significantly affect the difference in the bare energies of the ground state N and Δ . The latter is in contrast to the results of Ref. [4], and this difference may be due to the restricted set of intermediate states in that calculation, and also the use of model masses for the intermediate states, which moves thresholds away from their physical positions. It will be shown in the next section that in the present calculation the differences of the self energies of the negative-parity excited states do depend on the residual interactions between the quarks.

V. NON-STRANGE P -WAVE BARYON SPLITTINGS

The solution of Eq. (5) for the bare energies of the lowest-lying negative-parity N^* states with $J^P = 1/2^-$, corresponding to the states $N(1535)S_{11}$ and $N(1650)S_{11}$ seen in analyses of pion-nucleon scattering and photo-production data, is illustrated in Figure 5. It is clear that the inclusion of intermediate states involving both $N = 1$ negative-parity and $N = 2$ band positive-parity excited baryon states is crucial to the correct description of the bare masses of these states. Given the size of the self energies due to intermediate states involving $N = 2$ band baryons, it was necessary to calculate the effects on the bare masses of the presence of intermediate states involving $N = 3$ band highly-excited negative-parity baryon states. It is clear from Fig. 5 and Table I that these effects are roughly the same for both states, which means that their splitting is not strongly affected. Table I shows that, with Coulomb and contact interactions only in the qqq Hamiltonian which generates the wave functions used to evaluate the self energies, the bare mass of the $S_{11}(1650)$ state lies slightly below that of $S_{11}(1535)$. A similar situation arises for the pair of states $D_{13}(1520)$ and $D_{13}(1700)$. When this process is repeated for the other negative-parity non strange excited states, it is found that the bare energies required to fit the physical masses are not degenerate.

The pattern of splitting of the bare energies of these states and those of the nucleon and Δ ground states is shown in Figure 6. Interestingly, although inverted from the usual OGE quark-model expectations (where the predominantly spin-3/2 state lies above the predominantly spin-1/2 state), the bare mass splitting required to fit the physical masses of the two $N1/2^-$ states $N(1535)S_{11}$ and $N(1650)S_{11}$ is considerably smaller (about one third) than the physical

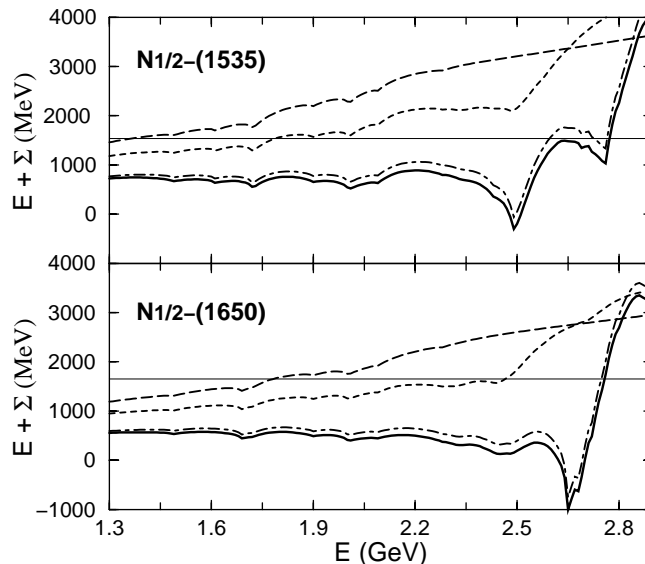


FIG. 5: Sum of bare and self energies for the P -wave excited states $N1/2^-(1535)$ (upper panel) and $N1/2^-(1650)$ (lower panel), for $\alpha = 0.5$ GeV and with Coulomb and contact interactions only in H_{qqq} . Curves are labeled as in Fig. 3. Note Eq. (5) is solved where the curves intersect the horizontal solid lines at 1535 and 1650 MeV, respectively.

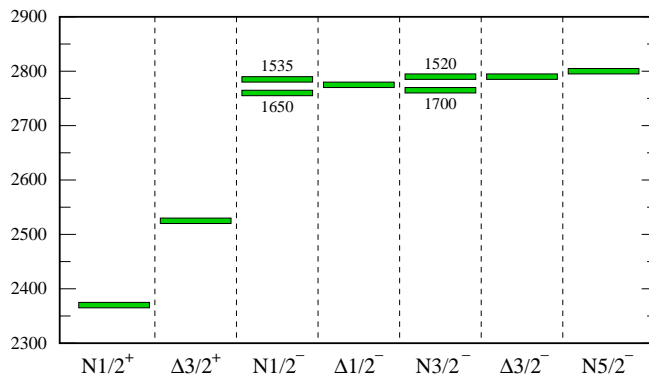


FIG. 6: Bare energies (in MeV) of ground and P -wave excited state non-strange baryons required to fit their masses from analyses of data, calculated using wave functions which are eigenfunctions of $H_0 + H_{\text{Coulomb}} + H_{\text{contact}}$, with harmonic-oscillator size parameter $\alpha = 0.5$.

mass splitting, similar to what was found in the Δ -nucleon ground state system. Put another way, this means that the corrections to the mass from the self energies resemble the splittings which arise from one-gluon exchange or other spin-dependent contact interactions. Note also that there are some effects which resemble tensor or spin-orbit interactions, such as the small bare mass splitting between the $\Delta1/2^-$ and $\Delta3/2^-$ states.

It is interesting to determine whether the bare mass spectrum required to fit the masses from analyses of scattering data depends on the presence of configuration mixing effects in the wave functions used to determine the $BB'M$ vertices and so the self energies. A simple way to test this, adopted here, is to include one-gluon-exchange (OGE) tensor interactions in the qqq Hamiltonian used to calculate the wave functions (and model masses) of the intermediate baryon states. In anticipation of the results of this calculation, one could argue that changes in the wave functions of the ground states from tensor interactions are at the level of a few percent. These changes are unlikely to have large effects on the bare masses of the nucleon and Δ ground states, because most of their self energies arises from intermediate states which include ground-state and low-lying negative-parity baryons.

The self energies due to ground state intermediate baryons will likely be unaffected. Although it is known [19] that large mixings of the negative-parity states arise from tensor interactions, the self energies of the nucleon and Δ ground states due to these intermediate states should also largely be unaffected, because these states are close to degenerate on the scale of the mass splitting between the ground states and the negative-parity excited states. Mixings, therefore, will shift strength around between individual intermediate states, but in this case the energy denominators in the

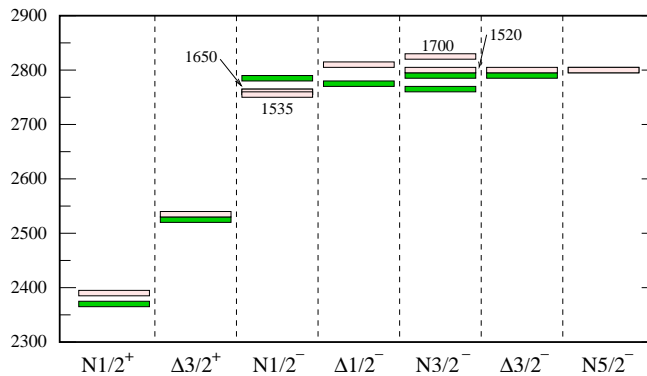


FIG. 7: Bare energies (in MeV) of ground and P -wave excited state non-strange baryons required to fit their masses from analyses of data, calculated using wave functions which are eigenfunctions of $H_0 + H_{\text{Coulomb}} + H_{\text{contact}} + H_{\text{tensor}}$ (lightly shaded boxes), compared to those calculated using eigenfunctions of $H_0 + H_{\text{Coulomb}} + H_{\text{contact}}$ (dark-shaded boxes, see Fig. 6), with harmonic-oscillator size parameter $\alpha = 0.5$.

loop integrals in Eq. (2) are roughly the same for each intermediate state.

This will not be true of the self energies of negative-parity excited states, where the mixings will have substantial effects on the vertex functions, and the energy denominators for $B \rightarrow B'M \rightarrow B$ can differ due to the proximity in mass of the initial and intermediate states. The effects on the bare masses of including these mixings are shown in Figure 7, where the spectrum of bare masses calculated with tensor mixings in the wave functions is contrasted to that from Fig. 6. The bare mass splitting between the ground state Δ and nucleon is slightly reduced, but there are substantial changes in the bare masses of the negative-parity states, and in the splitting between the average bare mass of the ground state and negative-parity excited states. The bare masses of the two $N1/2^-$ states are now in the usual order, if almost degenerate, and the same is true of the two $N3/2^-$ states. There is a small negative splitting which resembles a spin-orbit splitting between the bare masses of the $\Delta3/2^-$ (1700) and $\Delta1/2^-$ (1620) states which, interestingly, has the opposite sign to that expected in the OGE quark model of Ref. [12].

VI. COMPARISON OF BARE MASS AND MODEL SPECTRA

Given the substantial size of the splittings induced by baryon self energies demonstrated above, it can be argued that it should not be possible to fit the spectrum by ignoring them. Instead, it will be necessary to (i) postulate a qqq Hamiltonian, (ii) use the resulting wave functions to calculate the self energies which result from it, (iii) use these to find the bare baryon energies corresponding to the physical masses, and (iv) check to see whether the splittings between these bare energies match those between the model masses resulting from step (i). This process may need to be iterated.

However, prior calculations of the baryon spectrum which ignore the self energies seem able to roughly fit the physical masses, with some noticeable exceptions. As has been demonstrated above and in work by other authors [1, 3, 4, 5], this may be because the splittings in the bare masses often act in the same direction as spin-dependent contact interactions between the quarks. Figure 8 and Table IV show the comparison between the model masses resulting from the one-gluon exchange Hamiltonian described in Sec. IIIC and the corresponding bare masses. This comparison represents the second iteration in this process, where the strength of the one-gluon-exchange interaction has been reduced to roughly fit the $\Delta - N$ bare-mass splitting calculated consistently with the corresponding wave functions.

It is obvious from Fig. 8 that the string tension (or light quark mass) used in Ref. [12] and adopted here is not able to fit the splitting in the average bare masses of the ground states and negative-parity excited states. (This splitting does not change significantly with the addition of intermediate states involving $N = 3$ band baryons, as illustrated in Table I). It is possible to self-consistently fit the splitting of the Δ and nucleon bare masses, which is a non-trivial result, as pointed out in Ref. [4]. Figure 8 shows that the resulting model qqq Hamiltonian, if calculated with a consistent tensor interaction, gives model splittings which resemble those of the consistently calculated bare masses, with some differences. Although beyond the scope of the present work, given the sensitivity of the results for bare masses to the presence of the tensor interactions demonstrated above, and the results of previous calculations [1, 5, 6], it will be interesting to self-consistently calculate the effects of spin-orbit interactions.

TABLE IV: Spectrum in MeV of bare energies calculated using wave functions with $H_{qqq} = H_0 + H_{\text{Coulomb}} + H_{\text{contact}} + H_{\text{tensor}}$, and the corresponding model masses. Self energies are calculated using a full set of intermediate states with excited baryons up to $N_{\text{max}} = 3$. Both spectra have been normalized to reproduce the mass of $\Delta(1232)$ by adjusting an overall additive constant.

physical state	E^0	model mass
$N \frac{1}{2}^+(938)$	1087	1082
$\Delta \frac{3}{2}^+(1232)$	1232	1232
$N \frac{1}{2}^-(1535)$	1453	1500
$N \frac{1}{2}^-(1650)$	1457	1572
$\Delta \frac{1}{2}^-(1620)$	1507	1570
$N \frac{3}{2}^-(1520)$	1495	1506
$N \frac{3}{2}^-(1700)$	1520	1606
$\Delta \frac{3}{2}^-(1700)$	1495	1569
$N \frac{5}{2}^-(1675)$	1495	1584

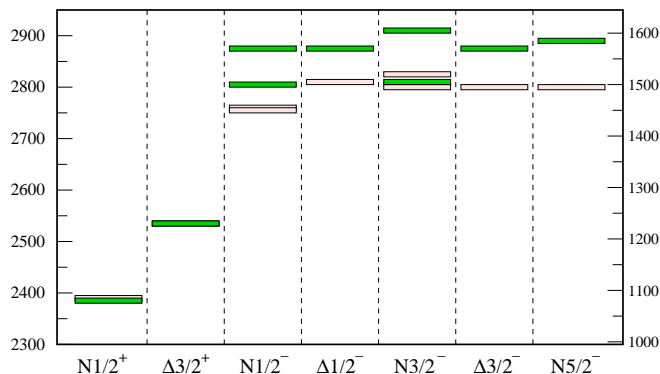


FIG. 8: Bare energies (in MeV) of ground and P -wave excited state non-strange baryons required to fit their masses from analyses of data, calculated using wave functions which are eigenfunctions of $H_0 + H_{\text{contact}} + H_{\text{tensor}}$ (lightly shaded boxes, left scale) with harmonic-oscillator size parameter $\alpha = 0.5$, compared to model masses (in MeV) from the same Hamiltonian (dark-shaded boxes, right scale). An overall constant has been added to the model masses to fit the $\Delta(1232)$ mass, and the two scales have been adjusted so that the two $\Delta(1232)$ masses coincide.

VII. CONCLUSIONS

The results shown above demonstrate that the sum over excited baryons in the intermediate state has converged to a stable splitting between the bare energies required to fit the physical ground-state nucleon and Δ masses. This requires the use of a full set of $SU(6)$ -related $B'M$ intermediate states, with excited baryons B' up to the top of the $N = 3$ band. This splitting is roughly 150 MeV, independent of the choice of inter-quark Hamiltonian H_{qqq} used to generate the wave functions, which affect the $BB'M$ vertices used to evaluate the self energies. In prior calculations of the baryon spectrum with one-gluon-exchange residual interactions between the quarks, this splitting has been used to fix the effective strength α_s of the short-range interactions between the quarks. This result implies that a reduced-strength one-gluon-exchange interaction should be employed in such calculations, designed to fit the roughly 145 MeV splitting between these states after the self-consistent correction for the self energies has been applied. Similar conclusions resulted from calculations which either did not use a full set of $SU(6)$ -related $B'M$ intermediate states or did not consider spatially excited intermediate baryons. The self-energy diagrams evaluated in this work include meson-exchange interactions between the quarks, and deal consistently with quark self energies from meson loops and the threshold effects of $B'M$ states. With this complexity in mind, it may still be possible to conclude that such meson-exchange effects are not the sole source of the $\Delta - N$ splitting.

Convergence of the sum over intermediate excited baryons of the bare energies required to fit the P -wave non strange baryon spectrum has been demonstrated using this same set of $B'M$ states, although in contrast to the situation with the ground states, $N = 2$ band states make important contributions. This resolves a problem with convergence found in a prior calculation, and points to the importance of taking into account the structure of the intermediate-state mesons, and the suppression of the creation of quark pairs with high relative momentum, as adopted previously by other authors. Interestingly, these bare energies are not degenerate, and also depend substantially on the inter-quark

Hamiltonian used to generate the wave functions which go into the calculation of the $BB'M$ vertices in the self energies. This means that it is necessary to use in the evaluation of these self energies a self-consistent calculation of both the spectrum, with its corresponding wave functions, and the strong vertices. The use in such calculations of unmixed oscillator wave functions to represent states whose physical masses enter into the positions of thresholds is inconsistent, and is likely to lead to erroneous results.

These observations suggest the use of a reduced-strength one-gluon exchange interaction to generate the wave functions used in the calculation of the vertices. The comparison of the resulting model spectrum with the bare energies required to fit the spectrum of ground and P -wave non strange baryons demonstrates that it is possible to understand the $\Delta - N$ splitting as partly due to self-energies and partly due to residual interactions between the quarks. Calculations which ignore the self energies overestimate the splitting between the ground states and the P -wave excited states. The comparison between the splittings of the P -wave baryon model masses and those of the resulting bare masses shows that tensor mixings are important, and this is likely to also be true of spin-orbit mixings.

An extension of the present work to the ground-state and negative-parity excited state Λ and Σ baryons along with a study of spin-orbit effects is currently in progress. This is of particular interest since intermediate states such as $\bar{K}N$ are known to have large effects on the masses and properties of several of the excited states, such as $\Lambda 1/2^-(1405)$. The mixing of both non-strange and strange baryon states with the same quantum numbers due to these $B'M$ intermediate states, ignored here for simplicity, is also under investigation, as are the effects of self-energies on positive-parity excited initial baryon states (such as the Roper resonance). This will likely require the inclusion of a second band of positive-parity excited states. At the same time the effects of excited meson states, presumed small for the reasons outlined above, should be checked by examining self energies due to the lightest orbitally excited mesons and ground-state baryons.

Acknowledgments

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APPENDIX A: TRANSITION AMPLITUDE

All of the details of the calculation of the strong decay transition amplitudes used in the present work are given elsewhere [20, 21], including the full form of the various components of the decay amplitude [14]. A summary of the main results used in their evaluation is given below.

The final form of the transition amplitude is

$$\begin{aligned}
A_{B \rightarrow B'M} &= \frac{6\gamma}{3\sqrt{3}} (-1)^{J_B + J_{B'} + \ell_B + \ell_{B'} - 1} \sum_{J_\rho, s_B, s_{B'}} \hat{J}_\rho^2 \hat{s}_B \hat{L}_B \hat{s}_{B'} \hat{S}_{B'} \hat{L}_{B'} \left\{ \begin{matrix} S_B & L_\rho & s_B \\ \ell_B & J_B & L_B \end{matrix} \right\} \left\{ \begin{matrix} L_\rho & S_\rho & J_\rho \\ \frac{1}{2} & s_B & S_B \end{matrix} \right\} \\
&\times \left\{ \begin{matrix} S_{B'} & L_\rho & s_{B'} \\ \ell_{B'} & J_{B'} & L_{B'} \end{matrix} \right\} \left\{ \begin{matrix} L_\rho & S_\rho & J_\rho \\ \frac{1}{2} & s_{B'} & S_{B'} \end{matrix} \right\} (-1)^{\ell + \ell_B + J_M - L_M - S_M} \mathcal{F}(BB'M) \mathcal{R}(BB'M) \\
&\times \sum_{S_{B'M}} (-1)^{s_B - S_{B'M}} \begin{bmatrix} J_\rho & 1/2 & s_{B'} \\ 1/2 & 1/2 & S_M \\ s_B & 1 & S_{B'M} \end{bmatrix} \sum_{L_{B'M}} (-1)^{L_{B'M}} \begin{bmatrix} s_{B'} & \ell_{B'} & J_{B'} \\ S_M & L_M & J_M \\ S_{B'M} & L_{B'M} & J_{B'M} \end{bmatrix} \\
&\times \sum_L \hat{L}^2 \left\{ \begin{matrix} s_B & \ell_B & J_B \\ L & S_{B'M} & 1 \end{matrix} \right\} \left\{ \begin{matrix} S_{B'M} & L_{B'M} & J_{B'M} \\ \ell & J_B & L \end{matrix} \right\} \varepsilon(\ell_{B'}, L_M, L_{B'M}, \ell, \ell_B, L, k_0). \tag{A1}
\end{aligned}$$

Here

$$\mathbf{J}_B = \mathbf{L}_B + \mathbf{S}_B = \ell_B + \mathbf{s}_B, \tag{A2}$$

with

$$\begin{aligned}
\mathbf{L}_B &= \mathbf{L}_{\lambda_B} + \mathbf{L}_{\rho_B} \equiv \ell_B + \mathbf{L}_{\rho_B}, \\
\mathbf{S}_B &= \mathbf{S}_{\rho_B} + 1/2, \tag{A3}
\end{aligned}$$

and

$$\mathbf{s}_B = \mathbf{J}_{\rho_B} + \mathbf{1}/2 = \mathbf{L}_{\rho_B} + \mathbf{S}_{\rho_B} + \mathbf{1}/2, \quad (\text{A4})$$

with similar definitions for B' . The first four $6-j$ symbols of Eq. (A1) are necessary for transforming from the usual angular momentum basis for the baryons, given by Eq. (A3), to the basis of Eq. (A4), which is the more convenient one for evaluating the transition amplitude. Here L , $L_{B'M}$ and $S_{B'M}$ are internal summation variables, $\mathcal{F}(BB'M)$ is the flavor overlap for the decay, and $\mathcal{R}(BB'M)$ is the overlap of the wave functions in the ρ coordinates in the initial and final baryons.

The purely “spatial” part of the transition amplitude is

$$\begin{aligned} \varepsilon(\ell_{B'}, L_M, L_{B'M}, \ell, \ell_B, L, k_0) &= \mathcal{J}(B) (-1)^{L_{B'M}} \frac{1}{2} \frac{\exp(-F^2 k_0^2)}{G^{\ell_B + \ell_{B'} + L_M + 4}} N_B N_{B'} N_M \\ &\times \sum_{\ell_1, \ell_2, \ell_3, \ell_4} C_{\ell_1}^{\ell_{B'}} C_{\ell_2}^{L_M} C_{\ell_3}^1 C_{\ell_4}^{\ell_B} (x - \omega_1)^{\ell_1} (x - \omega_2)^{\ell_2} (x - 1)^{\ell_3} x^{\ell_4} \\ &\times \sum_{\ell_{12}, \ell_5, \ell_6, \ell_7, \ell_8} (-1)^{\ell_{12} + \ell_6} \frac{\hat{\ell}_5}{\hat{L}} \begin{bmatrix} \ell_1 & \ell'_1 & \ell_{B'} \\ \ell_2 & \ell'_2 & L_M \\ \ell_{12} & \ell_6 & L_{B'M} \end{bmatrix} \begin{bmatrix} \ell_3 & \ell'_3 & 1 \\ \ell_4 & \ell'_4 & \ell_B \\ \ell_7 & \ell_8 & L \end{bmatrix} \\ &\times \left\{ \begin{matrix} \ell & \ell_{12} & \ell_5 \\ \ell_6 & L & L_{B'M} \end{matrix} \right\} B_{\ell_1 \ell_2}^{\ell_{12}} B_{\ell_3 \ell_4}^{\ell_5} B_{\ell'_1 \ell'_2}^{\ell_6} B_{\ell'_3 \ell'_4}^{\ell_7} B_{\ell'_5 \ell'_6}^{\ell_8} \\ &\sum_{\lambda, \mu, \nu} D_{\lambda \mu \nu}(\omega_1, \omega_2, x) I_\nu(\ell_5, \ell_6, \ell_7, \ell_8; L) \left(\frac{\ell'_1 + \ell'_2 + \ell'_3 + \ell'_4 + 2\mu + \nu + 1}{2} \right)! \\ &\times k_0^{\ell_1 + \ell_2 + \ell_3 + \ell_4 + 2\lambda + \nu} / G^{2\mu + \nu - \ell_1 - \ell_2 - \ell_3 - \ell_4}. \end{aligned} \quad (\text{A5})$$

In this expression, $\mathcal{J}(B)$ is the Jacobian factor, and N_B is a normalization coefficient for the wave function of initial baryon B .

The term $\sum_{\lambda, \mu, \nu} D_{\lambda \mu \nu}(\omega_1, \omega_2, x) I_\nu(\ell_5, \ell_6, \ell_7, \ell_8; L)$ arises from writing the product of the associated Laguerre polynomials and exponentials of the hadron wave functions (here $\mathbf{q}_B \equiv \mathbf{p}_{\lambda_B}$, with a similar definition for the daughter baryon)

$$\begin{aligned} &L_{n_{\lambda_B}}^{\ell_B} e^{-A^2 q_B^2/2} L_{n_{\lambda_{B'}}}^{\ell_{B'}} e^{-B^2 q_{B'}^2/2} L_{n_{\lambda_M}}^{L_M} e^{-C^2 q_M^2/2} \\ &\equiv \sum_{\lambda, \mu, \nu} D_{\lambda \mu \nu}(\omega_1, \omega_2, x) e^{-A^2 q_B^2/2} e^{-B^2 q_{B'}^2/2} e^{-C^2 q_M^2/2}. \end{aligned} \quad (\text{A6})$$

When the substitutions $\mathbf{q}_B = x\mathbf{k} + \mathbf{q}$, $\mathbf{q}_{B'} = (x - \omega_1)\mathbf{k} + \mathbf{q}$, $\mathbf{q}_M = (x - \omega_2)\mathbf{k} + \mathbf{q}$ are made, and the integrals over \mathbf{k} (the momentum of the final baryon) and \mathbf{q} are evaluated, the expression above results with I_ν a purely geometric factor.

In Eqs. (A1) and (A5),

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \hat{c} \hat{f} \hat{g} \hat{h} \hat{i} \begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix} \quad (\text{A7})$$

where

$$\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}$$

is the $9-j$ symbol, and $\hat{J} = \sqrt{2J+1}$.

In Eq. (A5)

$$\begin{aligned} x &= (B^2 \omega_1 + C^2 \omega_2 + f^2) (A^2 + B^2 + C^2 + f^2)^{-1}, \\ F^2 &= \frac{1}{2} \left[A^2 x^2 + B^2 (x - \omega_1)^2 + C^2 (x - \omega_2)^2 + f^2 (x - 1)^2 \right], \\ G^2 &= \frac{1}{2} (A^2 + B^2 + C^2 + f^2). \end{aligned} \quad (\text{A8})$$

ω_1 and ω_2 are ratios of various linear combinations of quark masses. In addition,

$$C_{\ell_1}^{\ell} = \sqrt{\frac{4\pi(2\ell+1)!}{(2\ell_1+1)![2(\ell-\ell_1)+1]!}},$$

$$B_{\ell_1\ell_2}^{\ell} = \frac{(-1)^{\ell}}{\sqrt{4\pi}} \hat{\ell}_1 \hat{\ell}_2 \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{A9})$$

and $\ell'_1 = L_{B'} - \ell_1$, $\ell'_2 = L_M - \ell_2$, $\ell'_3 = 1 - \ell_3$, $\ell'_4 = L_B - \ell_4$.

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